The Galactic Cosmic-ray Ionization Rate

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Abstract. This pedagogical note investigates the origin of low energy cosmic rays (below 1 GeV/nucleon). It is these cosmic rays that dominate the ionization of the translucent clouds.

1 Low Energy Cosmic Rays

Cosmic rays are charged particles and so are deflected by magnetic fields. The relativistic equation of motion of a particle with charge $q = Ze$ in a static magnetic field of strength B is

$$\frac{d\mathbf{p}}{dt} = \frac{q}{c}\mathbf{v} \times \mathbf{B}. \tag{1}$$

Since the Lorentz force is always perpendicular to the motion of the particle no energy is gained and thus the Lorentz factor $\gamma = (1 - \beta^2)^{-1/2}$ is constant; here $\beta = v/c$. Thus the equation is the same as the Newtonian version and the solution is a circle with "gyro" frequency, $\omega_g = qB/(\gamma mc)$ and gyro-radius, $r_g = cp_\perp/(qB)$ where $p_\perp = \gamma m v_\perp$ is the momentum perpendicular to the magnetic field direction. Rigidity of a charged particle, $q = Ze$, is defined as

$$P \equiv \frac{cp_\perp}{Ze}. \tag{2}$$

The numerator has the dimensions of energy and the denominator is charge. Thus the dimensions of P is volt. With this definition, $r_g = P/B$. For a given field configuration, all particles with a given rigidity have the same gyro-radius. With useful normalization,

$$r_g = \frac{2.2}{Z}\Big(\frac{p_\perp}{10\,\mathrm{GV/c}}\Big)\Big(\frac{B}{1\,\mu\mathrm{G}}\Big)^{-1}\ \mathrm{AU}. \tag{3}$$

The Earth's magnetic field can be approximated by a dipole. At the magnetic equator (which passes over Kanyakumari at the tip of the Indian peninsula, for instance) the magnetic field strength is about 0.6 Gauss (60 μT for physicists) only particles with rigidity of 30 GV can reach ground.

At Earth's orbit (1 AU) the typical physical parameters of the undisturbed solar wind are: wind velocity of $350\,\mathrm{km\,s^{-1}}$ and particle density of $10^3\,\mathrm{cm^{-3}}$ (composition similar to that of the corona). The mean magnetic energy density is $10^{-10}\,\mathrm{erg\,cm^{-3}}$ (corresponding to a field strength of mean energy of $50\,\mu\mathrm{G}$). The mean energy density is $4\times10^{-9}\,\mathrm{erg\,cm^{-3}}$, dominated by protons. Clearly the solar wind is not only collisional-less but also dominated by particle energy. At 1 AU, the gyro-radius for a particle with rigidity of 1 GV is about a solar radius.

Incidentally, the gyro-radius reappears in an argument by the physicist Michael Hillas, a pioneer of cosmic ray research. The argument is utterly simple: a particle which leaves the acceleration region will cease getting accelerated! Ergo, the maximum energy of particles is determined by the gyro-radius, hence the inclusion of the "Hillas" diagram in every textbook on cosmic rays.

Low energy cosmic rays interact strongly with matter via ionization. They are very bad for living creatures but essential for the chemistry of diffuse clouds. The solar-wind follows the Sun's 11-year cycle. When the Sun is active the solar wind (carrying magnetic field and particles) is stronger and as a result low energy (say, <1 GeV/nucleon) cosmic rays are deflected away – as can be seen from Figure 1.

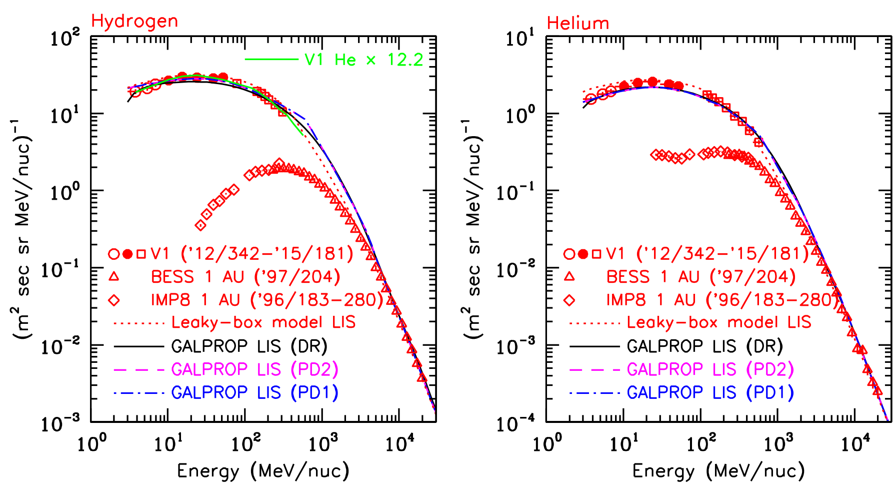

Figure 1: Measurements of cosmic rays (protons, alpha particles) at low energies by a variety of missions: Voyager-1 between 2012–2015, balloon flight BESS and IMP8 (geocentric orbit, period 12 days). From Cummings et al. (2016; [csh+16]). Note the exclusion of low energy cosmic rays from the vicinity of Earth by the solar wind.

In order to measure the low energy cosmic ray intensity we need detectors in the ambient interstellar medium, that is outside the shock created by the Solar wind. The Voyager missions in the outskirts of the Solar system[1] offer unique opportunities to measure the low energy cosmic ray spectrum (Cummings et al. 2016; [csh+16]; see Figure 1).

Homework Problem: Assume that the Earth's magnetic field is a pure dipole of mean strength 1 Gauss. Develop a numerical integration package and determine the orbit of particles with different rigidity injected say one AU away.

2 Loss Suffered by Cosmic Rays

2.1 Loss from Ionization

The topic of ionization losses due to cosmic rays was a major topic of investigation in the early days of cosmic ray research (in this era cosmic ray research was synonymous with the field of particle physics). The ionization by cosmic rays lay at the heart of emulsion detectors, cloud chambers and scintillation detectors. Proton beam therapy (involving protons in the range 50–270 MeV/nucleon) shows continued interest in this topic.

Longair (2011; [L11]; Chapter 5) provides an excellent introduction to this vast topic. Consider an electron (mass m_e) that is bound to an atom ("target"). Further assume that the electron does not move during the entire period of encounter with a high energy particle (mass, M, charge Z, velocity v, kinetic energy E; "beam"). The electron receives a net momentum primarily centered the closest encounter $2Ze^2/bv$ where b is the impact parameter.[2] This momentum is perpendicular to the linear trajectory of the cosmic ray. Integrating over b the rate of loss in energy of the cosmic ray per unit length traversed is

$$-\frac{dE}{dx} = \frac{4\pi Z^2 e^4 n_e}{m_e v^2} \ln\Big(\frac{b_{\rm max}}{b_{\rm min}}\Big) \tag{4}$$

where n_e is the number density of electrons.

The duration over which the electron derives must of the momentum impulse is $2b/v$. If this duration is longer than the orbital period, $2\pi/\omega_0$, of the electron then it would adjust its orbit adiabatically and thus not get ionized. Thus, we can it reasonable to have the interaction time $2b/v \ll 2\pi/\omega_0$. Classically the minimum impact parameter is obtained by requiring the encounter impart the maximum energy obtainable in a head-on collision: $2m_e v^2$. Thus, $b_{\rm min} \approx \pi v/\omega_0$. The maximum linear momentum gained is $2m_e v$

[1] Circa September 2019: Voyager-1 or V1 is 141 AU and V2 is 117 AU from the Sun. For reference, the last contact with Pioneer-10 was at 80 AU and 40 AU for Pioneer-11.

[2] The interaction results in transferring a certain amount of momentum. The resulting energy is $\propto m^{-1}$ and as a result we need concern ourselves only with electron exciation.

and thus from a quantum perspective, $b_{\rm min} \approx \hbar/(2m_e v)$. The two approaches converge at a velocity

$$\frac{1}{Z\alpha}\beta = \frac{137}{Z}\beta \tag{5}$$

where α is the fine structure constant. So, for cosmic rays with $\beta < Z/137$ we should use the classical limit and quantum otherwise. Since we are interested in fast moving particles we will henceforth ignore the classical limit and proceed with the quantum approach. The energy loss equation now becomes

$$-\frac{dE}{dx} = \frac{4\pi Z^2 e^4 n_e}{m_e v^2} \ln\Big(\frac{2\pi m_e v^2}{\hbar\omega_0}\Big). \tag{6}$$

We recognize that the ionization potential is $I = \frac{1}{2}\hbar\omega_0$. Generalizing to atoms with many electrons we should replace I by $\bar{I}$. The final equation is

$$-\frac{dE}{dx} = 4\pi\frac{Z^2 e^4 n_e}{m_e v^2} \ln\Big(\frac{m_e v^2}{\bar{I}}\Big) \tag{7}$$

For particles moving at relativistic speeds the electrostatic field has to be transformed with Lorentz equations. The electron experiences a stronger electrical field but for a shorter time (and also a magnetic field, but this is not with any significant consequence). The maximum energy that can be transferred to the electron is limited by energy-conservation laws (the "Bhabha correction"):

$$T_{\rm max} = \frac{2m_e\beta^2\gamma^2}{1 + 2\gamma(m_e/M) + (m_e/M)^2}. \tag{8}$$

The net result is the famous Bethe equation:

$$-\frac{dE}{dx} = 4\pi\frac{Z^2 e^4 n_e}{m_e v^2}\Big[\ln\Big(\frac{2\gamma^2\beta^2 m_e c^2}{\bar{I}}\Big) - \beta^2 - \frac{1}{2}\ln[1 + 2\gamma(m_e/M) + (m_e/M)^2]\Big]. \tag{9}$$

The Bethe equation is correct to $O(z^2)$ where z is the nuclear charge of the target atom. Successive corrections, $O(z^3)$ (Barkas & Anderson), $O(z^4)$ (Bloch) and correcting for the shell architecture of the electrons of the target atom leads to the Bethe-Bloch formula.

There are two key points that are worth noting. First the ionization loss depends only on Z, the nuclear charge and v, the velocity of the projectile. The energy loss scales as v^{-2} or E^{-1} at low velocities,[3] reaching a minimum at $\gamma\beta \approx 2$ (when the projectile's kinetic energy equals its rest mass), followed by a slow rise $\propto \log(E)$. Next, since $\bar{I}$ is in the log, the loss rate is relatively insensitive to the nature of the target. However, as discussed above there

[3]The rapid loss of energy as the particle slows down is the basis of proton therapy; search and study "Bragg peak".

Target	A	B	C	D
H	0.28	1.15	0.44	0.907
He	0.49	0.62	0.13	1.52
N_2	3.82	2.78	1.80	0.70
O_2	4.77	0.0	1.76	0.93
Ne	1.63	0.73	0.31	1.14
Ar	3.85	1.98	1.89	0.89
Kr	5.67	5.50	2.42	0.65
Xe	7.33	11.1	4.12	0.41

Table 1: Model parameters for proton ionization cross-sections of various elements (from Rudd et al. 1985 [rkm+85].

is some dependence on z. Finally, experiments show that the ionizations results in electrons with typical energy of 34 eV (air), 36 eV (H) and 26 eV (Ar).

Consider a cosmic ray particle going through the interstellar medium of which H is the dominant atom. The loss per ionization is 36 eV (Dalgarno & Griffing 1955; [DG55]) plus 13.6 (the ionization potential of H I) which leads to $E_i \approx 50\,\mathrm{eV}$ loss per ionization. There are two principal channels of interaction for the ejected electron: interacting with a thermal electron or an H I atom. For largely neutral medium with ionization fraction $x < 0.01$ the former can be neglected (Spitzer & Tomosako 1968; [st68]). Collisions with H I atoms can be either elastic (the energy is not radiated away) or inelastic. The fractional amount of energy transferred in elastic collisions is small, $O(m_e/m_p)$. Next, the ejected electron can further ionize another H atom and also excite an H I atom leading to loss via Lyman lines. Once the energy of the electron falls below the 3/4 Rydberg (the $1 \rightarrow 2$ transition) then the electron will undergo elastic collisions. Spitzer & Tomosako (1968; [st68]) find $\langle E_h \rangle \approx 3.5\,\mathrm{eV}$.

2.2 Loss from Scattering Electrons

Read up Spitzer's 1962 plasma physics book.

3 Measurements & Models

Padovani, Galli & Glassgold (2009; [pgg09]) is an excellent starting point for the atomic physics of ionization of helium by fast ions and electrons (and charge exchange at low energies). Rudd et al. (1985; [rkm+85]) report measurements for ionization of HI by protons over the energy range 10–1,500 keV and He by protons over the range 1–5,000 keV.

They provide a fitting formula for the ionization cross section, σ_i, as a function of x, where $x = E/R$ with E being the energy of the beam particle and $R = 13.6\,\mathrm{erg}$ is the Rydberg:

$$\sigma_i = \frac{1}{\sigma_l^{-1} + \sigma_h^{-1}} \tag{10}$$

where the "low" and "high" energy fits are given by

$$\sigma_l = 4\pi a_0^2 C x^D \tag{11}$$

$$\sigma_h = 4\pi a_0^2 \Big[A \log(1+x) + B\Big] x^{-1} \tag{12}$$

where $a_0 = 0.529\,\text{Å}$(Bohr radius) and A, B, C and D are fitting parameters (see Table 1).

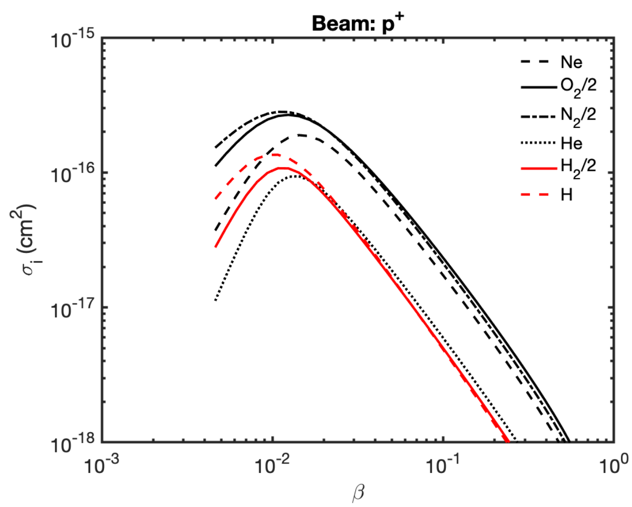

Figure 2: Model fits to measured cross-sections for ionization of various gases by fast moving protons (Rudd et al.; [rkm+85]). The measurements were undertaken for protons with energies between few keV to few MeV. The cross-sections for O_2, N_2 and H_2 were divided by two to account for two atoms. On average the cross-section of metals (i.e. other than H and He) is three times larger than that for HI.

In Figure 2 I plot the fits to experimental measurements of O_2, N_2, Ne, Kr and others. For the diatomic molecules I divide the cross-section by two to account for the fact that there are two atoms per target. As discussed earlier we do expect to see an increase in the ionization cross-section with z. Empirically, from Figure 2 I determine that the ionization cross-section for higher-z elements is, on the average, a factor of three larger than that of hydrogen.

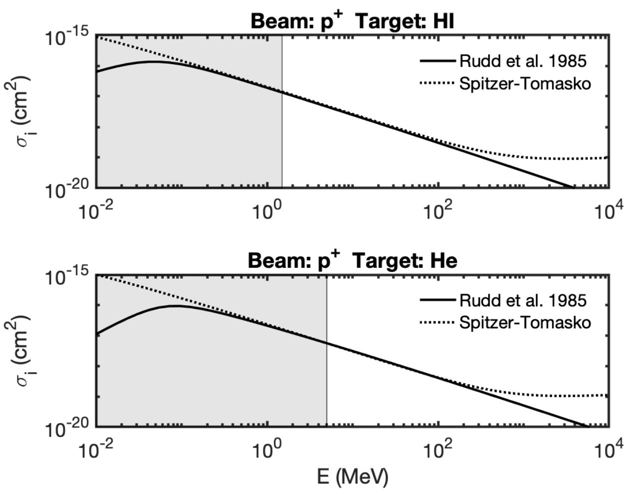

Figure 3: The run of the measured cross section of ionization (thick line; from Rudd et al.1985; [rkm+85]) by fast moving protons (H) and alpha particles (He) as a function of energy. The dotted line is the the Bethe formula but as provided by Spitzer & Tomasko (1968; [st68]). The grey zone is the energy range over which the experimental data was obtained. The fitting formula for the experimental data can be found in the text.

Spitzer & Tomasko (1968; [st68]) provide the following approximation for the Bethe-Bloch formula for ionization of HI by protons with energy $> 0.3\,\mathrm{MeV}$,

$$\sigma_i = 1.23 \times 10^{-20} \frac{Z^2}{\beta^2}\Big[6.20 + 2\log(\gamma\beta) - 0.43\beta^2\Big]. \tag{13}$$

The run of the measured σ_i against β and the Spitzer-Tomasko approximation is shown in Figure 3. In the same Figure I display the measurements of proton impact ionization of helium. The Spitzer-Tomasko formula provides a good fit for the measurements provided that Equation 13 cross-section is increased by 0.15. [I suppose it is because He has two electrons but this is compensated by a higher ionization potential of He relative to H].

4 Measurements of Cosmic Rays by Voyager-1

The local cosmic ray intensity, averaged over solid angle, is

$$J(E) \approx 1.8\Big(\frac{E}{\mathrm{GeV}}\Big)^{-\alpha} \text{ particles cm}^{-2}\text{ s}^{-1}\text{ ster}^{-1}\text{ GeV}^{-1} \tag{14}$$

where E is the energy per nucleon and $\alpha \approx 2.7$. Around $10^{15}\,\mathrm{eV/nucleon}$ (the "knee") the slope steepens to ≈ 3 and E is the kinetic energy of the particles. Cosmic rays above $10^{18}\,\mathrm{eV/nucleon}$ are called as ultra-high energy cosmic rays.

In 2012, the particle detectors aboard the Voyager-1 mission showed an increase in the low energy channels. It is widely regarded that the spacecraft had crossed the "heliopause" (which, for an astronomer, would be the contact discontinuity surface). Cummings et al. (2016; [csh+16]) report the V1 measurements of low energy particles (electrons, protons, alpha, nuclei) over the period 2012–2015 (Figure 4). They fit the observed spectrum to several Galactic propagation models. Using `Vizier` I downloaded GALPROP model "DR" (Table 13 of this paper).

As with any radiation field, the spectral particle density is related to the mean intensity through the relation

$$n(E) = 4\pi \frac{J}{v(E)} \tag{15}$$

where $v(E)$ is the velocity of particle with energy E. The ionization rate of hydrogen by cosmic rays with nuclear charge Z is

$$\zeta_H(Z) = 4\pi(1+\phi_s)\int J(E)\sigma_i(E,Z)dE \tag{16}$$

and the (kinetic) energy density is

$$\epsilon_{\rm CR} = \int n(E)EdE \tag{17}$$

where $1+\phi_s$ is a factor that accounts for ionization caused by the ejected electrons. For reasons discussed briefly towards the end of §2.1 this factor depends on the ionization state of the gas (see Chapter 13 of Draine 2011; [D11])). For now, we will, following Cummings et al. (2016; [csh+16]) set $1+\phi_s = 1.5$.

Following Cummings et al. (2016; [csh+16]) I used the Spitzer-Tomasko formula (Equation 13) and was able to reproduce their quoted results: $\zeta_H(p^+) = 0.76\times10^{-17}\,{\rm s}^{-1}$, $\zeta_H(\alpha) = 0.24\times10^{-17}\,{\rm s}^{-1}$ and $\zeta_H(Z>2) = 0.29\times10^{-17}\,{\rm s}^{-1}$. Thus, the cosmic ray ionization of H due to nuclei is thus $\zeta_H(Z\geq1) = 1.28\times10^{-17}\,{\rm s}^{-1}$.

The Cummings et al. (2016; [csh+16]) ionization rate does not include ionization of atoms other than hydrogen by cosmic rays. The analysis that I carried out in the previous section (§3 shows that, relative to the ionization cross section of hydrogen, that of helium is 15% larger and that of higher Z atoms is 3 times larger. We assume the following fraction, by number: 90% (H), 9% (He) and 1% ($Z>2$). Then, ionization of He is $0.09\times1.15\times\zeta_H(Z\geq1)$ and that of all other atoms is $0.01\times3\times\zeta_H(Z\geq1)$. Thus, the ionization of atoms other than H amounts to $0.13\zeta_H(Z\geq1)$ and the total stands at $1.13\zeta_H(Z\geq1)$.

Cummings et al. (2016) find that energetic electrons contribute $\zeta_H(e^-) = 0.23\times10^{-17}\,{\rm s}^{-1}$. Including the ionization of He and other elements by electrons will increase the electron ionization rate to at least $1.1\zeta_H(e^-)$.

Thus, the total ionization rate is $1.13\zeta_H(Z \geq 1)$ plus $1.1\zeta_H(e^-)$ which amounts to $1.7 \times 10^{-17}\,\mathrm{s}^{-1}$. This is ten times smaller than that inferred from the chemistry of diffuse clouds.

The energy density of cosmic-ray protons and alpha particles is $\epsilon(p^+) = 0.72\,\mathrm{eV\,cm^{-3}}$ and $\epsilon(\alpha) = 0.18\,\mathrm{eV\,cm^{-3}}$, respectively. After including all other species (Li-Ni, e) the total energy density is $\epsilon = 1.56\ \mathrm{eV\,cm^{-3}}$. In detail, I found that protons and helium nuclei with energy less than 67 MeV/nucleon produce half of their ionization rates, 10% of ionization is contributed by protons with $E > 1\,\mathrm{GeV}$ and alpha particles with $E > 0.7\,\mathrm{GeV}$ (see Figure 4). In contrast, half the H energy density is contributed protons with energy less than 2.6 GeV and half of the He energy density is contributed helium nuclei with energy less than 2.1 GeV. Inversely, 1% of the energy density is carried by 3–67 MeV particles and 20% of energy density is carried by 3–0.7 GeV protons. So there is a big difference in the energy of the particles which ionize and particles which contribute to the energy density.

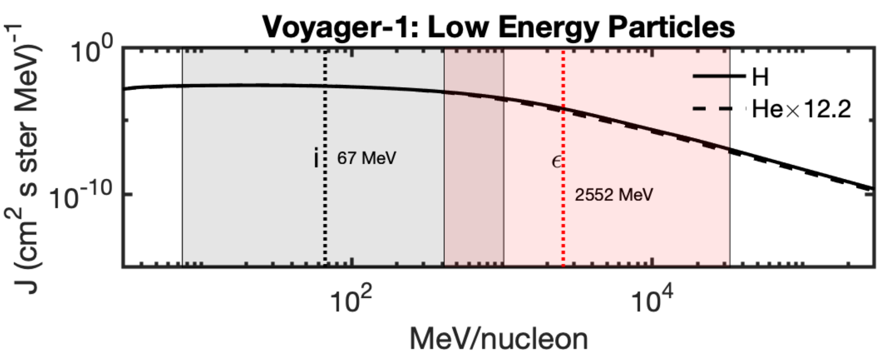

Figure 4: (Top): The estimated interstellar cosmic ray energy spectrum obtained from Voyager 1 in the period from 2012 (when there was a dramatic increase in the low energy cosmic rays; crossing the heliopause) through 2015. The spectrum shown above is the data fitted to the GALPROP DR model (Table 13 from Cummings et al. 2016; [csh+16]). I downloaded Table 13 using `Vizier`. Following Cummings et al. 2016 [csh+16] the He intensity has been multiplied by 12.2. Reading from left to right: the vertical dotted (also marked with "i") line marks the median energy of protons causing ionization of H atom whereas the next vertical line (also marked with "ϵ") marks the median energy of protons contributing to the cosmic ray energy density. The gray region marks the 10%-90% of the protons which contribute to ionization whereas the pink region the same but for energy density. Notice bulk of the ionizing protons arise from the "flat" portion of the cosmic ray intensity whereas cosmic rays around the turnover energy contribute to the energy density.

Homework: Cummings et al. (2016; [csh+16]) compute the ionization of only H by energetic electrons. Compute the ionization of all atoms dues to energetic electrons.

5 Astrophysical Measures of CR ionization

The start of gas-ion chemistry requires ionized hydrogen. FUV photons are absorbed at the surface. Low energy cosmic rays were therefore invoked to account for ionic molecules (e.g. OH^+, H_3^+) and other products of gas ion chemistry. Neufeld & Wolfire (2017; [nw17]) provide a nice summary of the inferences of cosmic ray ionization within diffuse atomic (tracers: OH^+, H_2O^+, ArH^+) and diffuse (H_3^+) and dense molecular clouds (HCO^+). They conclude that $\Gamma_{\rm CR}$ to be $2\times10^{-16}\,{\rm s}^{-1}$ in diffuse atomic and diffuse molecular clouds but decreasing to $10^{-17}\,{\rm s}^{-1}$ in dense molecular clouds. Low frequency radio recombination lines offer another promising way to investigate translucent clouds (the interface between surfaces of molecular clouds) – see Oonk et al. 2017 [ovs+17]).

6 Resolving the Discrepancy

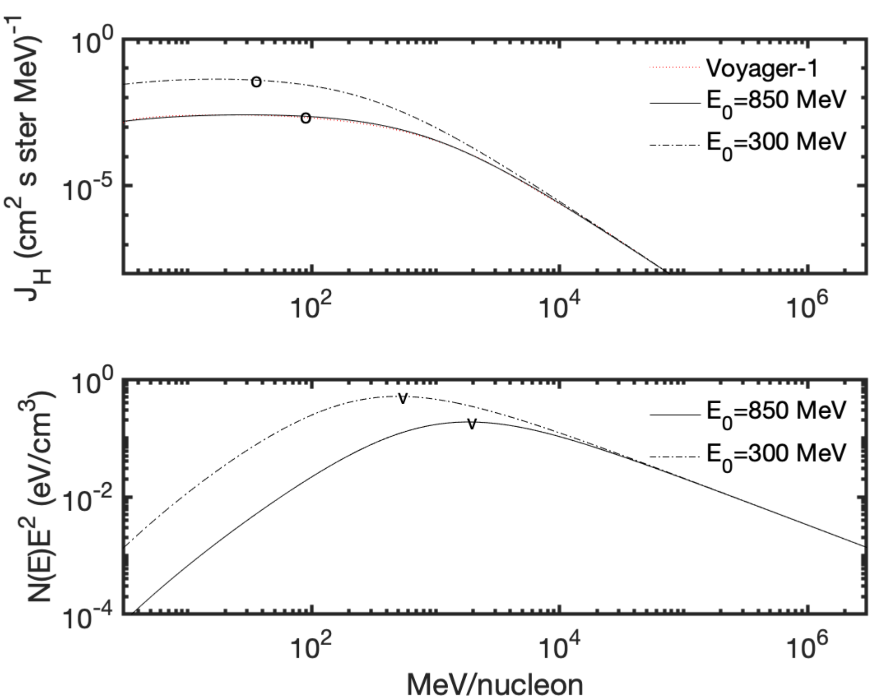

Figure 5: TOP: The cosmic ray intensity spectrum for the Voyager data (black line) and the Spitzer-Tomasko formula with $E_0 = 850\,{\rm MeV}$ (dotted red line which coincides with the black line) and $E_0 = 300\,{\rm MeV}$. The circles mark the median energy of ionizing protons. BOTTOM: $N(E)E^2$ for the two models discussed above. The letter"v" mark the peak of each curve.

Clearly, there is a major discrepancy between the *measurements* of low energy cosmic rays by Voyager-1 at *one location* and the astronomical *inferences* towards *many lines of sight* and that too at several Galactocentric radii. It seems downright parochial to accept the simplest possibility, namely, we live in a cosmic-ray poor region of the Galaxy. Nonetheless, let us consider this possibility and study the consequence. In particular, we will assume that the energy at which the cosmic ray intensity "turns over" varies with location.

To explore this possibility I adopt the parametric model for the proton intensity given in Spitzer & Tomasko (1968; [st68]):

$$J_{\rm ST}(E) = \frac{A}{(E_0+E)^\alpha}\frac{1}{1+E_c/E}\ {\rm cm^{-2}\,sec^{-1}\,MeV^{-1}\,ster^{-1}} \tag{18}$$

Setting $A = 5\times10^5$, $E_0 = 850\,{\rm MeV}$ and $E_c = 3\,{\rm MeV}$ provides an excellent fit to the Voyager-1 data (Figure 5). We note that E_c is a minor parameter and so we keep it fixed. However, E_0 is a major parameter. Varying E_0 changes the ionization rate. For instance, setting E_0 to 300 MeV results in $\zeta_H(p^+)$ of $9\times10^{-17}\,{\rm s^{-1}}$ which is about 11 times the corresponding value computed for the Voyager-1 data (see Figure 5). The corresponding energy density in protons is $\epsilon(p^+) = 2\,{\rm eV\,cm^{-3}}$ which is about three times larger than for the Voyager-1 data. Assuming the same factor holds for alpha particles and other nuclei we find $\epsilon = 3\times1.56 \approx 4.7\,{\rm eV\,cm^{-3}}$. This makes cosmic rays the dominant component in the energetics of the diffuse medium. For instance the magnetic field strength and thermal pressure corresponding to ϵ is $14\,\mu$G and $\mathcal{P} = 5.4\times10^4\,{\rm cm^{-3}\,K}$.

Next, if indeed $E_0 \sim 300\,{\rm MeV}$ over most of the Galaxy then the total cosmic ray power is $\Gamma_{{\rm CR},n}E_iM_{\rm H}/m_H + \Gamma_{{\rm CR},e}M_{\rm H+}/m_H$ where $E_i \approx 50\,{\rm eV}$ (see discussion towards end of §2.1) is the mean energy per ionization (including the ionization potential energy) $M_{\rm H}$ is the mass in H I and $M_{\rm H+}$ is the mass ionized hydrogen. We exclude the molecular gas which is $\approx 1.2\times10^9\,M_\odot$ from this exercise. From Draine (2011; [D11]; Table 1.2) we have $M_{\rm H} \approx 4\times10^9\,M_\odot$ and $M_{\rm H+} = 1.5\times10^9\,M_\odot$. With the above cosmic ray rate the cosmic ray power is $[0.8, 1.6]\times10^{41}\,{\rm erg\,s^{-1}}$ for H and electron losses, respectively. The total is $2.4\times10^{41}\,{\rm erg\,s^{-1}}$.

The standard analysis is that supernovae shocks are responsible for cosmic rays. Assuming one SN with energy release of 10^{51} erg every century leads to a mechanical power of $3\times10^{41}\,{\rm erg\,s^{-1}}$. Thus, if (1) the cosmic ray spectrum in most regions of the Galaxy had the same form as the Voyager-1 spectrum but (2) with $E_0 = 300\,{\rm MeV}$ to account for $\Gamma_{\rm CR}$ of $2\times10^{-16}\,{\rm atom^{-1}\,s^{-1}}$ then the cosmic ray power that is needed is almost equal to that produced by supernovae. So, on energetic grounds, we conclude that the model with $E_0 = 300\,{\rm MeV}$ is not viable.

The next simplest conclusion is that there are two sources of cosmic rays: high energy cosmic rays ($\ll$ GeV and up) and low energy cosmic rays ($\ll 1\,{\rm GeV}$). This suggestion is not new (e.g. Webber 1983). The high energy spectrum is summarized by Equation 14 and arises from supernovae shocks. In contrast, the low energy spectrum results from local sources (e.g. shocks arising from stellar winds, bow shocks, numerous low energy explosions such as novae). In particular stellar winds, especially from massive stars, inject about 25% of power, relative to SNe. Thus, a reasonable conclusion is that the cosmic ray ionization rate is a factor of 10 higher only in star-forming regions but has the Voyager-1 value in most of the Galaxy.

`Some thoughts:` The "range" – the column density over which the cosmic ray loses most of its energy – decreases with the speed of the particle. For instance, the range of a 2-MeV proton is $3 \times 10^{-3}\,\mathrm{gcm}^{-2}$ or a column density of $1.8 \times 10^{21}\,\mathrm{cm}^{-2}$ (Spitzer & Tomosako 1968; [st68]). Low energy cosmic rays will be scattered by irregularities in the interstellar magnetic field. The net distance they can travel is the geometric mean between rectilinear path (600 pc, assuming mean density of 1 atom cm^{-3}) and the typical scattering length. [NEED TO FIGURE OUT WHAT IS THIS LENGTH]. Thus, the sources for low-energy cosmic rays would have to be numerous both to satisfy the range requirements and also to suppress strong fluctuations in the cosmic ray rate.